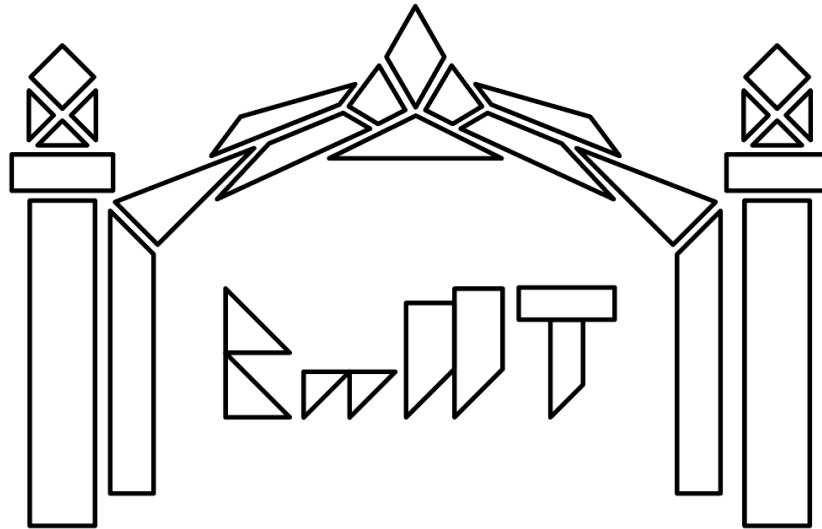


Berkeley mini Math Tournament 2024

Puzzle Round Problems



April 14, 2024

Time limit: 15 minutes for reading; 60 minutes for solving.

Instructions: For this test, you work in teams of up to five to solve a set of puzzles. You will have 15 minutes to read the rules of the Puzzle Round and to strategize. Afterwards, you will have 60 minutes to solve the puzzles. Only submit the set of answer sheets for grading.

No calculators.

Introduction

Welcome to Shapelink! This companion document will be available to you throughout the entire duration of the round. It's highly recommended that everyone on your team work through all provided examples. Remember to submit your team's solutions on the provided answer sheet.

A reminder about difficulty: Puzzles are arranged in increasing difficulty **by variant only**. Feel free to skip to different sections, where puzzles may be easier. Partial credit may be given, so submit any progress you have made on a puzzle. Solutions to puzzles (except for the **Flow** variant) must be unique, so if you find multiple solutions, check to make sure that you've understood the rules fully.

Basic Shapelink

Drawing Polygons

A blank Shapelink puzzle will look something like this:

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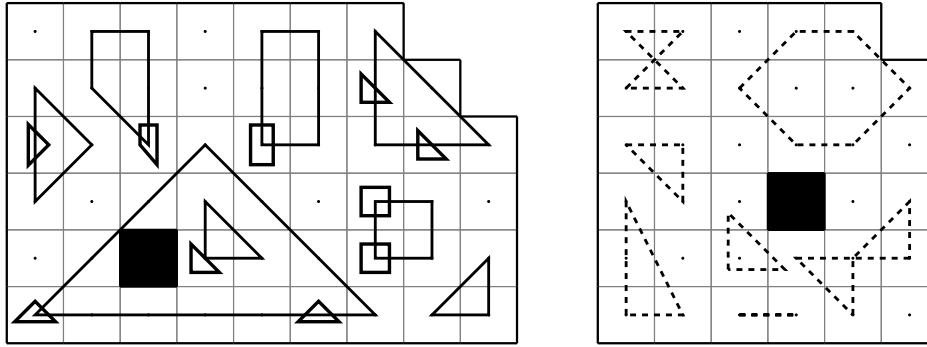
Broadly speaking, the puzzle consists of a grid of square cells with different types of clues in individual cells. To solve the puzzle, polygons must be drawn in the grid to satisfy all of the given clues. There are several rules about how individual polygons must be drawn:

1. **Centered vertices:** Every vertex of a polygon must be in the **center** of a grid cell.
2. **Eight directions:** Every edge of a polygon must be drawn horizontally, vertically, or at a 45° angle to one of those four cardinal directions. (Think of a queen in chess; see examples below.)
3. **Non-degenerate:** Polygons should not be degenerate (they should not take up zero area).
4. **Four sides:** A polygon may have at most **four** vertices/edges.

Additionally, there are two rules about how polygons may interact with each other:

1. **No intersections:** Polygons should not intersect each other or themselves: two edges of distinct polygons should not share points, and two edges of the same polygon may only intersect at a vertex.
2. **No cloning:** Identical polygons (*clones*) are forbidden to appear in the same grid. Two polygons are *clones* if they are congruent AND oriented the same way. (Intuitively, clones look exactly the same.)

Some examples of polygons drawn in the grid are shown on the next page. (Ignore the smaller shapes inside of individual cells; those are *shape* clues, which we will learn about later.)



The left grid shows polygons drawn correctly in the grid. Some things to note:

- The two right triangles covering matching shape clues are NOT clones; their sizes are different, so they aren't congruent. Also, the two congruent right triangles are not clones as they have different orientations.
- Polygons can be drawn inside of other polygons.
- Edges may touch the borders of the grid and blacked out regions, as long as they don't cross fully over them.
- Polygons may be drawn without covering shape clues.

The right grid shows polygons drawn incorrectly; each rule is broken by at least one polygon. Refer back to the six rules listed above to figure out why all of these polygons are illegally drawn for one reason or another. An important note: the two triangles with right angles at their top right corners are clones of each other, which is why they are illegal. However, if the bottom-right one of these triangles were removed from the grid, the other triangle would be legal to draw (along with the triangle that the bottom-right triangle is intersecting).

Understanding Clues

Understanding shape clues requires a few important definitions.

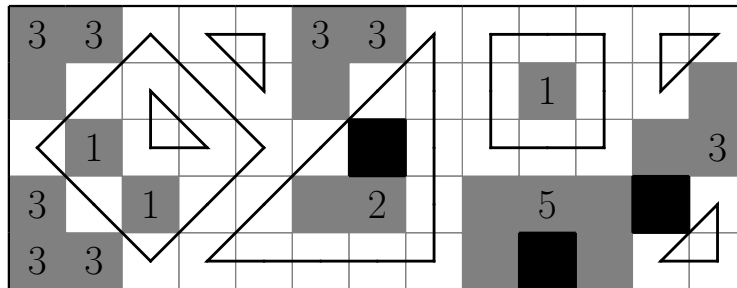
1. **Filled cells:** If an edge of a polygon passes through **the center** of a grid cell, then that cell is *filled* by that polygon (passing through just a corner does NOT count).
2. **Empty cells:** If a grid cell is not filled, it is *empty*.
3. **Shape clues:** A **shape** clue is satisfied if a matching polygon fills the cell the clue is in. Intuitively, a polygon matches a shape clue if it's a larger version of that clue: the polygon should be of the same dimensions and have the same orientation, but can be a different size.
4. **Mystery clues:** A **mystery** clue indicates that *some* polygon fills the cell the clue is in, but does not specify which polygon that is.

Diagrams are provided on the last page of the rules to precisely define what polygons match a given shape clue. **Other polygons may be drawn besides the ones shown in the diagrams.** The diagrams ONLY show every shape clue that is given during the round, and polygons that satisfy them. To see an example of polygons matching shape clues in the grid, take a look back at the diagram at the top of this page.

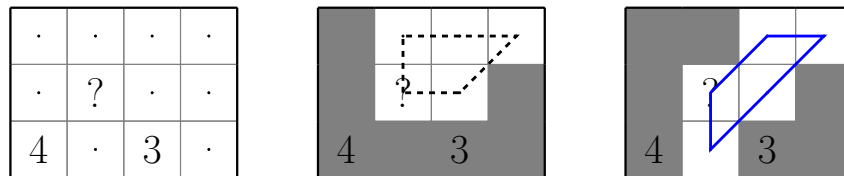
We need one more definition for the last type of clue, the *hole* clue.

1. **Holes:** A *hole* is a group of empty cells that are all connected orthogonally (horizontally or vertically, but not diagonally), and its *size* is the number of empty cells in the group.
2. **No bordering holes:** Two holes can NEVER touch orthogonally (touching diagonally is allowed). Holes never border any empty cells; they take up all the empty cells that they border.
3. **Hole clues:** The **hole** clues (numbers) give the size of the *hole* containing that cell (this means any cell with a hole clue must be empty).

Here are some diagrams to explain how these work:

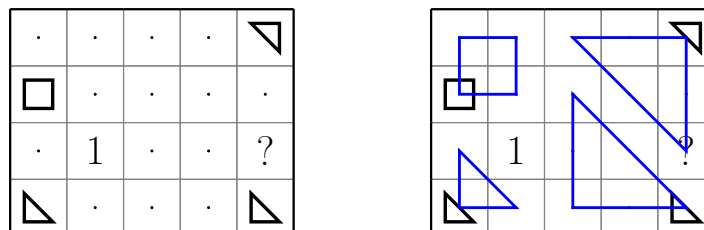


Cells belonging to holes are shaded gray. (Shading in holes can earn partial credit points, but it's not necessary if the full solution is written down.) All hole clues in the above diagram are satisfied. Note that one hole is allowed to satisfy multiple hole clues. Blacked-out regions (*voids*) do not count as cells, and so do not count towards the size of a hole.



In this small puzzle, the middle solution is incorrect because the hole clues do not correspond to holes of size 4 and 3. They instead correspond to one hole of size 7, meaning that neither clue is correct. This comes from the fact that holes take up all empty cells that is available to them. The solution on the right is the correct solution, with the two holes not touching each other orthogonally.

Let's go back to the puzzle from the start for an example of a fully solved **Basic** Shapelink:

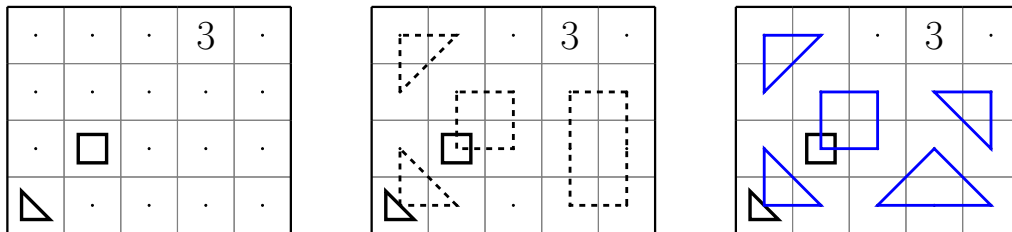


Variant 1: Tangram

Tangram adds one simple rule adjustment to the **Basic** ruleset.

1. **Full covering:** Every empty cell must be part of a hole that has a hole clue.

So, besides those empty cells clued by *hole* clues (numbers), every cell in the grid must be filled by some polygon. That's it! Here's an example puzzle (left), an incorrect solution (middle), and the correct solution (right). The incorrect solution has an unclued hole making it wrong.

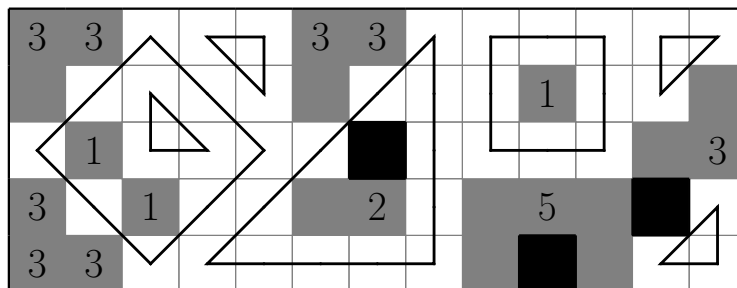


Variant 2: Ghosts

Ghosts adds two rules to the **Basic** ruleset.

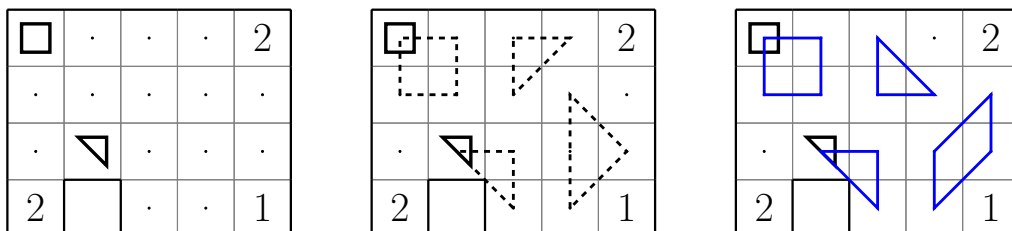
1. **Full covering:** Every empty cell must be part of a hole that has a hole clue.
2. **No ghosts:** The no cloning rule now also applies to holes: holes that are identical (*ghosts*) cannot appear in the same grid.

To explain the second rule more, let's look back at the holes diagram from earlier:



All 1 clues are always clones of each other, so there can only ever be one 1 clue in a Ghosts puzzle. The diagram is written so that holes with the same number of 3 clues in them are *ghosts*, but holes with a different number of 3 clues are okay to coexist in the same grid. Generally, rotations of the same hole are allowed to coexist. Otherwise, there are no other clones in the grid above.

Here's an example puzzle (left) with an incorrect solution (middle) and correct solution (right). The middle solution is wrong because the 2 holes are clones.

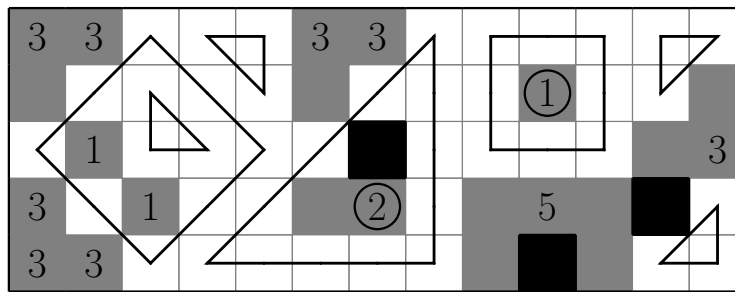


Variant 3: Districts

Districts introduces a new type of clue to the **Basic** ruleset, along with one additional rule.

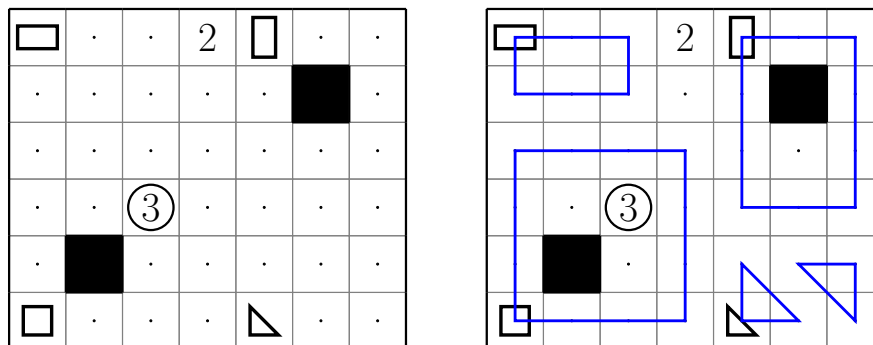
1. **Interiors:** A *hole* is called an *interior* if it occurs inside of a polygon. The *size* of the interior is the same as the size of the hole.
2. **Interior clues:** An *interior clue* looks like a number inside of a circle. It indicates the size of the interior containing that cell. **It also indicates that the polygon containing that interior should not contain any polygons.**
3. **Region covering:** This is NOT the same as **Tangram** rules. For **Districts**, each cell in the grid must be either (1) *filled* by a polygon edge, (2) part of a **clued** hole, or (3) part of some *interior* (NOT necessarily clued).

We'll take one last look back at the holes diagram to understand things a little better:



Numbers that are circled correspond to valid interior clues. One important thing to notice is that the two diagonally touching 1 clues are NOT valid interior clues, because of the triangle inside of the square. Also, like with the hole diagram, the interior of size 2 does NOT include the void as part of its size, since the void is not considered a cell in the grid.

Here's an example puzzle to tie all these concepts together:

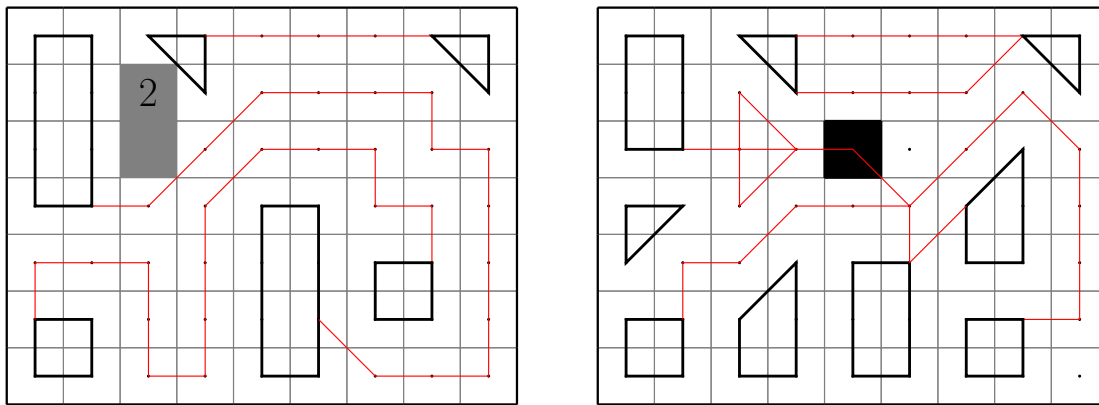


Variant 4: Flow

Flow builds on the **Basic** ruleset, except the **no cloning** rule is no longer in effect.

1. **Twins:** Each polygon must have *exactly one* clone (these two clones are called **twins**).
2. **Paths:** Each pair of **twins** must have *exactly one* **path** connecting them. A path is a sequence of edges (following the same rules as polygon edges) drawn in the grid, so that only consecutive edges share vertices and otherwise edges do not intersect each other. The first and last edge of the path must each have one vertex on the boundaries of a pair of twins.
3. **No path intersections:** Edges of distinct paths cannot intersect each other. Edges of paths cannot intersect any polygons (except the first and last edge, as described above).
4. **Path filling:** Edges of paths also fill cells as described in the basic rules; any cell with an edge of a path passing through its **center** is *filled* by that path. **Note:** Mystery clues cannot be satisfied by paths: they must be filled by polygons.
5. **Full covering:** Every empty cell must be part of a hole that has a hole clue.

For some intuition on what paths can and can't look like, here's two diagrams:

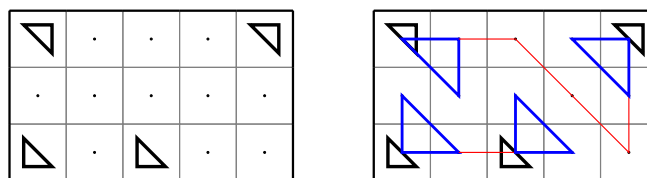


Left: Correct, Right: Incorrect

On the right, there are several problems. The path between the squares intersects the path between the rectangles. The path between the rectangles goes over a void, and is self-intersecting. The trapezoids are not connected by a path, and also one trapezoid is connected to a rectangle (which is not its twin). The triangle on the left has no twin in the grid. There are two paths between the twin triangles on the top (exactly one path should be drawn).

One final note: solutions to Flow puzzles are not unique. Shapes will be placed uniquely in the grid, and individual paths will have to cover the same group of cells, which may be done in several different ways. However, the broad strokes of the solution will always be the same.

Here is an example of a Flow puzzle for reference.



Puzzle Solving Checklist

In this section, we provide a short list of rules for each variant to help you check that you've solved a puzzle correctly. We refer to each rule by its title (1-3 bold words at the start). If you don't remember what a specific rule says, check the main rules again! If you want, tear off this last page of the rules to make it easier to use during the solving period.

Remember that the **Basic** rules apply to all puzzles (except for **no cloning**, which doesn't apply to **Flow**).

Remember to shade in clued holes for potential partial credit! (*Not necessary for fully solved puzzles*)

Basic

- **Centered vertices**
- **Eight directions**
- **Non-degenerate**
- **Four sides**
- **No intersections**
- **No cloning** (does not apply to **Flow**)
- **Shape clues**
- **Mystery clues**
- **Hole clues**
- **No bordering holes**

Tangram

- **Full covering**

Ghosts

- **Full covering**
- **No ghosts**

Districts

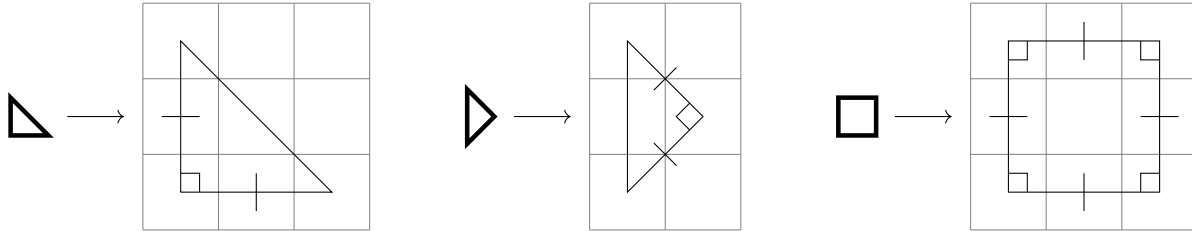
- **Interior clues**
- **Region covering**

Flow

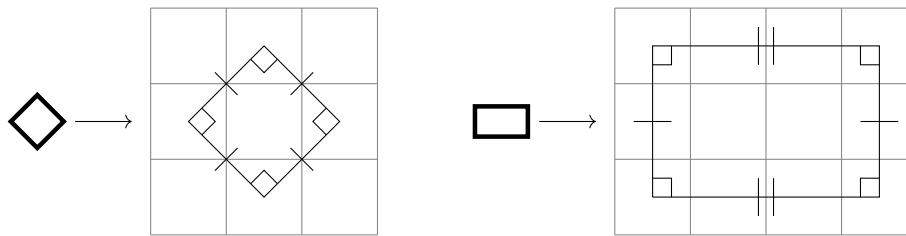
- **Twins**
- **Paths** (paths fill cells!)
- *Paths don't satisfy mystery clues*
- **No path intersections**
- **Full covering (Flow)**

Shape Clue Diagrams

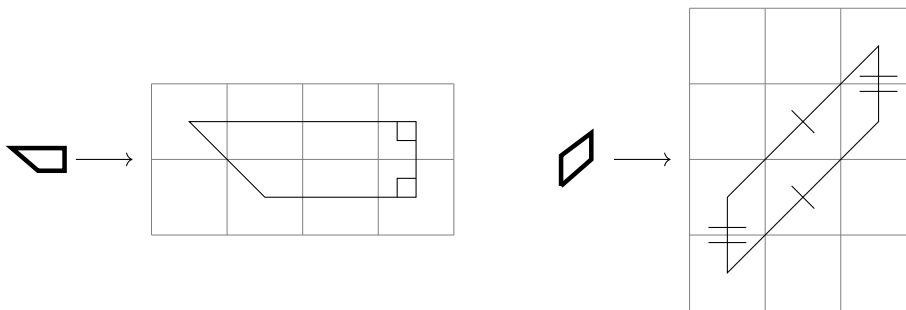
To reiterate: this is not a comprehensive list of all polygons that may be drawn in the grid. You may draw any polygon satisfying the 6 rules given in the **Drawing Polygons** section. Sides with equal numbers of tick marks through them must always be drawn with equal lengths in the grid.



For the rectangle clue, the side that appears longer in the shape clue should match in the drawn polygon. In particular, a “tall” rectangle shouldn’t be covered by a “wide” rectangle or a square; the vertical sides should be strictly longer than the horizontal sides.

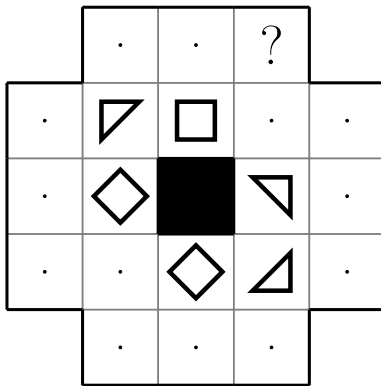


For the parallelogram clue, the diagonal side lengths can be drawn longer or shorter than the vertical ones; in general, relative side lengths do not matter for parallelogram clues, as long as opposite sides have equal lengths.

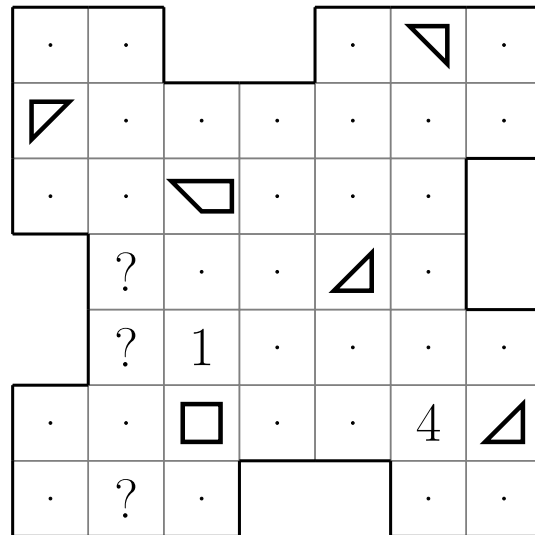


Basic

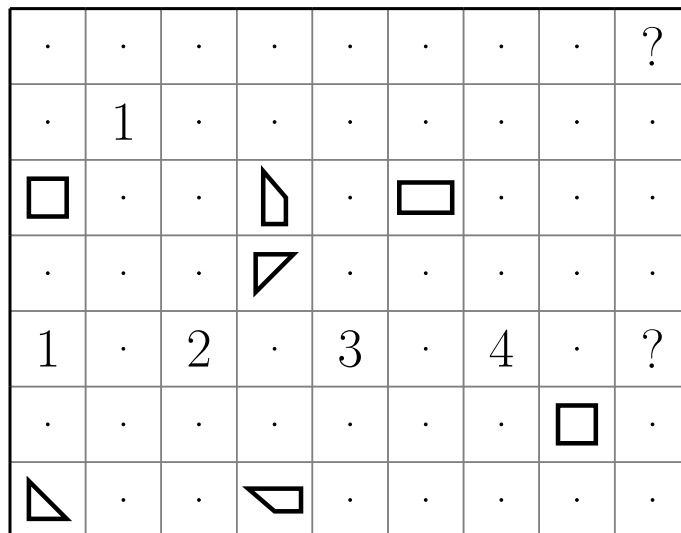
Basic 1 (12 points)



Basic 2 (16 points)

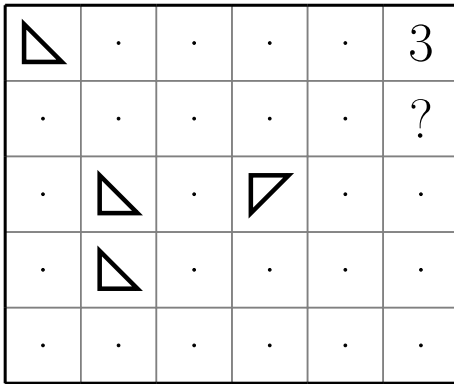


Basic 3 (20 points)

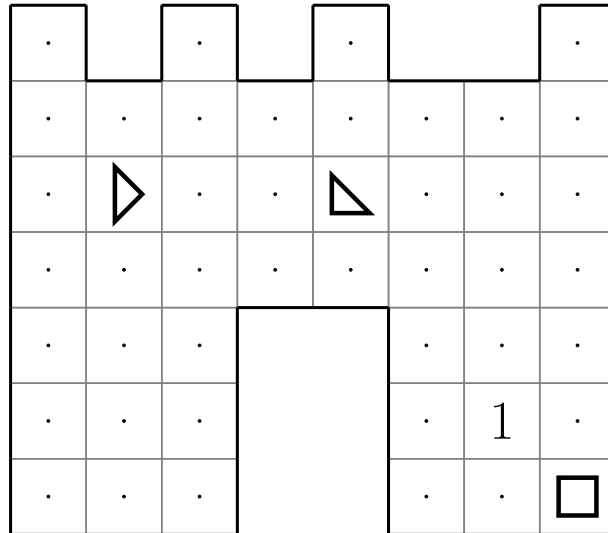


Tangram

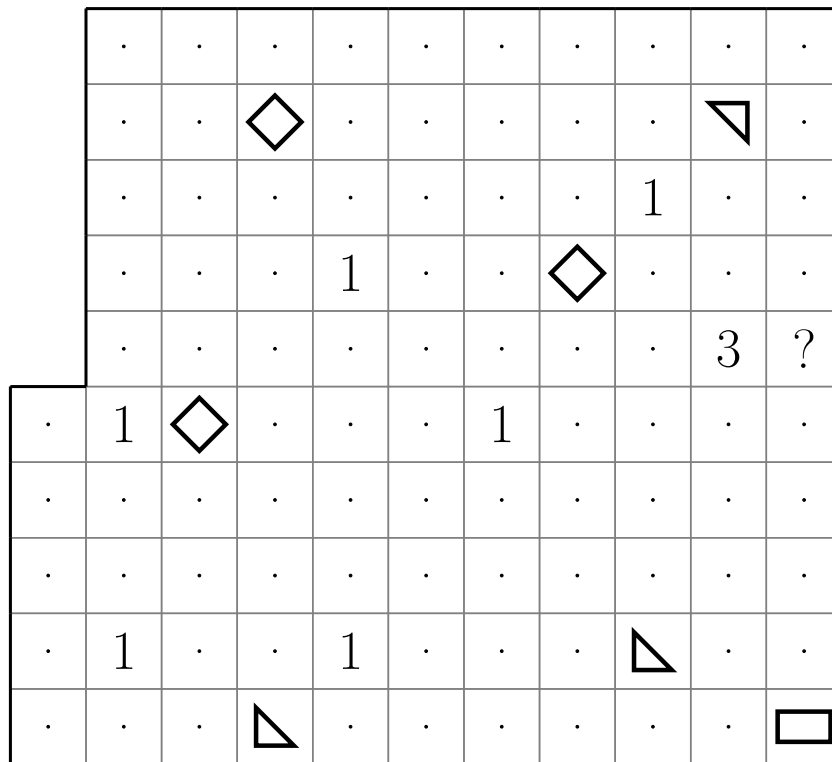
Tangram 1 (12 points)



Tangram 2 (16 points)



Tangram 3 (20 points)



Ghosts

Ghosts 1 (12 points)

3	.		3	.
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Ghosts 2 (16 points)

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					.	.	4	.
				
.	▽	.	.	2
.	4	?	□
.	?	4	.	.	△	.	.	.
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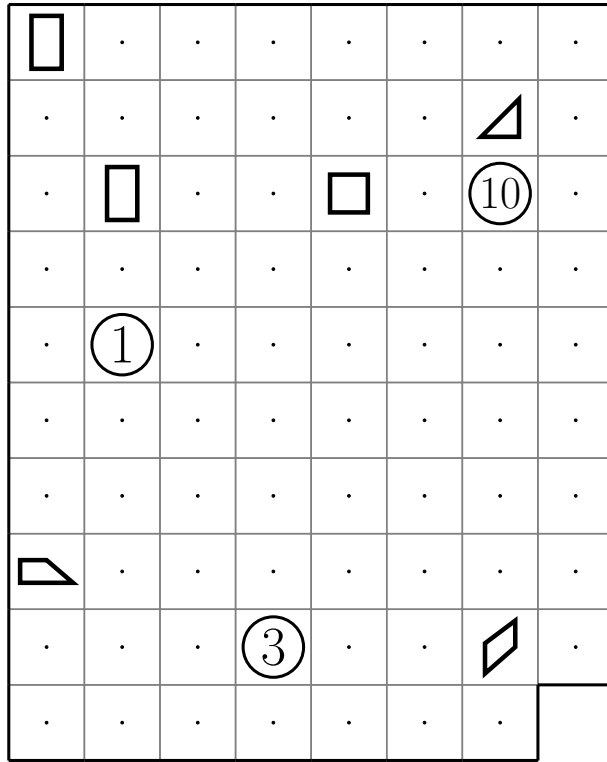
Ghosts 3 (20 points)

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	.	.	3	.	.	.	3
	3	.	.	.	3	.	.

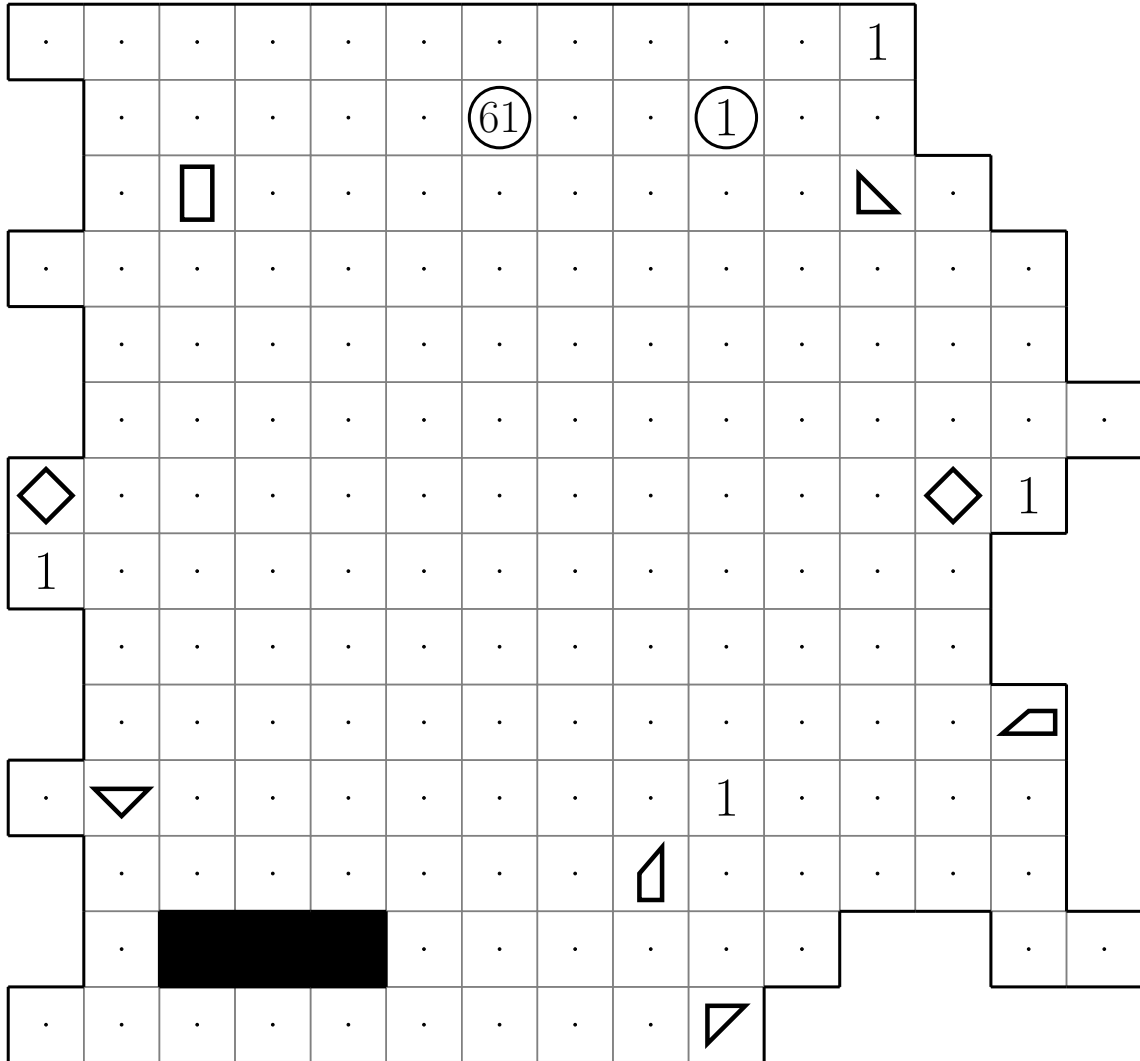
	.	▽	3	.	.	.	2
	.	.	▽
4	.	.	1

Districts

Districts 1 (12 points)



Districts 2 (16 points)



Districts 3 (20 points)

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Flow

Flow 1 (12 points)

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·	□	·	1	·	1	·	·	·
		1	·	1	·	1		
·	·	·	1	·	1	·	·	·
·	·	·	·	1	·	·	·	·
·	·	·	▷		·	·	·	·
□	·	·	·		·	·	□	·

Flow 2 (16 points)

▷	·	·	?	·	·	·	·	·	□
·	·	▷	·	·	·	1	▷	1	·
·	·	·	·	·	·	·	·	·	·
·	·	·	·	2	·	·	·	·	·
·	?	·	·	·	·	·	·	·	·
·	·	·	·	·	·	2	·	·	·
·	·	·	·	·	▷	·	·	?	·
□	·	·	·	·	·	·	·	·	·
·	1	·	1	·	·	·	·	·	·
·	·	·	·	·	·	▷	·	·	·

Flow 3 (20 points)

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·	8	·	·	·	·	·	·	·	·	·	·
·	·	·	2	·	·	·	·	·	▷	·	·
·	·	·	·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·	▷	·	·
1	·	10	·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·	▷	·	·
▷	·	·	▷	·	·	·	·	·	·	·	·
·	·	▷	·	·	·	·	·	1	·	·	·
▷	·	▷	·	·	·	▷	▷	·	1	·	·
·	·	2	·	·	·	·	·	·	·	▷	·
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