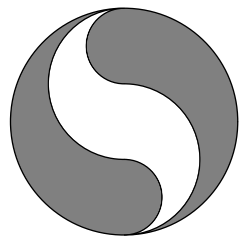
1. Billy the kid likes to play on escalators! Moving at a constant speed, he manages to climb up one escalator in 24 seconds and climb back down the same escalator in 40 seconds. If at any given moment the escalator contains 48 steps, how many steps can Billy climb in one second?

2. S-Corporation designs its logo by linking together 4 semicircles along the diameter of a unit circle. Find the perimeter of the shaded portion of the logo.



3. Two boxes contain some number of red, yellow, and blue balls. The first box has 3 red, 4 yellow, and 5 blue balls, and the second box has 6 red, 2 yellow, and 7 blue balls. There are two ways to select a ball from these boxes; one could first randomly choose a box and then randomly select a ball or one could put all the balls in the same box and simply randomly select a ball from there. How much greater is the probability of drawing a red ball using the second method than the first?

4. Let $ABCD$ be a square with side length 2, and let a semicircle with flat side CD be drawn inside the square. Of the remaining area inside the square outside the semi-circle, the largest circle is drawn. What is the radius of this circle?

5. Two positive integers m and n satisfy

$$\max(m,n) = (m-n)^2$$
$$\gcd(m,n) = \frac{\min(m,n)}{6}$$

   Find $\operatorname{lcm}(m,n)$.

6. Bubble Boy and Bubble Girl live in bubbles of unit radii centered at $(20,13)$ and $(0,10)$ respectively. Because Bubble Boy loves Bubble Girl, he wants to reach her as quickly as possible, but he needs to bring a gift; luckily, there are plenty of gifts along the x-axis. Assuming that Bubble Girl remains stationary, find the length of the shortest path Bubble Boy can take to visit the x-axis and then reach Bubble Girl (the bubble is a solid boundary, and anything the bubble can touch, Bubble Boy can touch too).

7. Given real numbers a, b, c such that $a + b - c = ab - bc - ca = abc = 8$. Find all possible values of a.

8. The **three-digit** prime number p is written in base 2 as p_2 and in base 5 as p_5, and the two representations share the same last 2 digits. If the ratio of the number of digits in p_2 to the number of digits in p_5 is 5 to 2, find all possible values of p.

9. An ant in the xy-plane is at the origin facing in the positive x-direction. The ant then begins a progression of moves, on the n^{th} of which it first walks $\frac{1}{5^n}$ units in the direction it is facing and then turns 60° degrees to the left. After a very large number of moves, the ant's movements begins to converge to a certain point; what is the y-value of this point?

10. If five squares of a 3×3 board initially colored white are chosen at random and blackened, what is the expected number of edges between two squares of the same color?

11. Let $t = (a, b, c)$, and let us define $f^1(t) = (a + b, b + c, c + a)$ and $f^k(t) = f^{k-1}(f^1(t))$ for all $k > 1$. Furthermore, a permutation of t has the same elements, just in a different order (e.g., (b, c, a)). If $f^{2013}(s)$ is a permutation of s for some $s = (k, m, n)$, where k, m, and n are integers such that $|k|, |m|, |n| \leq 10$, how many possible values of s are there?

12. Triangle ABC satisfies the property that $\angle A = a \log x$, $\angle B = a \log 2x$, and $\angle C = a \log 4x$ radians, for some real numbers a and x. If the altitude to side AB has length 8 and the altitude to side BC has length 9, find the area of $\triangle ABC$.

13. Let $f(n)$ be a function from integers to integers. Suppose $f(11) = 1$, and $f(a)f(b) = f(a + b) + f(a - b)$, for all integers a, b. Find $f(2013)$.

14. Triangle ABC has incircle O that is tangent to AC at D. Let M be the midpoint of AC. E lies on BC so that line AE is perpendicular to BO extended. If $AC = 2013, AB = 2014, DM = 249$, find CE.

15. Let $ABCD$ be a convex quadrilateral with $\angle ABD = \angle BCD$, $AD = 1000$, $BD = 2000$, $BC = 2001$, and $DC = 1999$. Point E is chosen on segment DB such that $\angle ABD = \angle ECD$. Find AE.

16. Find the sum of all possible n such that n is a positive integer and there exist a, b, c real numbers such that for every integer m, the quantity $\dfrac{2013m^3 + am^2 + bm + c}{n}$ is an integer.

17. Let $N \geq 1$ be a positive integer and k be an integer such that $1 \leq k \leq N$. Define the recurrence $x_n = \dfrac{x_{n-1} + x_{n-2} + ... + x_{n-N}}{N}$ for $n > N$ and $x_k = 1$, $x_1 = x_2 = ... = x_{k-1} = x_{k+1} = ... = x_N = 0$. As n approaches infinity, x_n approaches some value. What is this value?

18. Paul and his pet octahedron like to play games together. For this game, the octahedron randomly draws an arrow on each of its faces pointing to one of its three edges. Paul then randomly chooses a face and progresses from face to adjacent face, as determined by the arrows on each face, and he wins if he reaches every face of the octahedron. What is the probability that Paul wins?

19. Equilateral triangle ABC is inscribed in a circle. Chord AD meets BC at E. If $DE = 2013$, how many scenarios exist such that both DB and DC are integers (two scenarios are different if AB is different or AD is different)?

20. A sequence a_n is defined such that $a_0 = \dfrac{1 + \sqrt{3}}{2}$ and $a_{n+1} = \sqrt{a_n}$ for $n \geq 0$. Evaluate

$$\prod_{k=0}^{\infty} 1 - a_k + a_k^2$$