1. Every face of a cube is colored one of 3 colors at random. What is the expected number of edges that lie along two faces of different colors?

   Answer: 8

   Solution: The probability that the two faces bordering an edge are different colors is $\frac{2}{3}$. There are 12 edges in the cube, so by linearity of expectation, the expected number of such edges is $12 \cdot \frac{2}{3} = \boxed{8}$.

2. 6 people stand in a circle with water guns. Each person randomly selects another person to shoot. What is the probability that no pair of people shoots at each other?

   Answer: $\mathbf{\frac{1472}{3125}}$

   Solution: Use principle of inclusion exclusion.

   Alternative solution: start from person 0, and follow chain of people shot till it returns to someone who already fired. Do casework on length of that path.

   Case $n = 3$: $\frac{4}{5} \cdot \frac{1}{5} \cdot (\frac{3}{5}\frac{24}{25} + \frac{2}{5}(\frac{3}{5} + \frac{4}{25}))$

   $n = 4$: $\frac{4}{5} \cdot \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{24}{25}$

   $n = 5$: $\frac{4}{5}\frac{3}{5}\frac{2}{5}\frac{3}{5}$

   $n = 6$: $\frac{4}{5} \cdot \frac{3}{5} \cdot \frac{2}{5}\frac{1}{5}\frac{4}{5}$

   The answer is thus $\frac{440+576+360+96}{5^5} = \boxed{\frac{1472}{3125}}$.

3. Alice and Bob are playing rock paper scissors. Alice however is cheating, so in each round, she has a $\frac{3}{5}$ chance of winning, $\frac{2}{5}$ chance of drawing, and $\frac{2}{5}$ chance of losing. The first person to win 5 more rounds than the other person wins the match. What is the probability Alice wins?

   Answer: $\mathbf{\frac{243}{275}}$

   Solution: Markov chain with state being difference in score, then Gambler's ruin. Let a_k probability of Alice winning given current score difference is k. We have the recurrence $a_k = pa_{k+1} + qa_{k-1}$, with $a_5 = 1$, $a_{-5} = 0$. The characteristic polynomial is then $z = pz^2 - q$, which has roots $z_1 = 1$, $z_2 = \frac{q}{p}$, so $a_k = C_1 \cdot 1^k + C_2 \cdot \left(\frac{q}{p}\right)^k$. Solving the boundary conditions, we have $a_{-5} = C_1 + C_2\left(\frac{q}{p}\right)^{-5} = 0$, $a_5 = C_1 + C_2\left(\frac{q}{p}\right)^5 = 1$, so $C_2\left(\left(\frac{q}{p}\right)^k - \left(\frac{q}{p}\right)^{-k}\right) = 1$, so $C_2 = \frac{(q/p)^5}{(q/p)^{10}-1}$, $C_1 = \frac{-1}{(q/p)^{10}-1}$. We thus see that $a_0 = \frac{(q/p)^5-1}{(q/p)^{10}-1} = \frac{1}{(q/p)^5+1} = \frac{1}{(2/3)^5+16} = \boxed{\frac{243}{275}}$