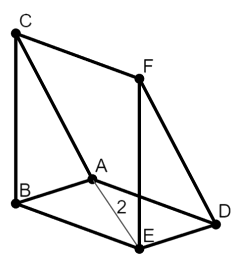
**Time limit:** 60 minutes.
**Instructions:** This test contains 10 short answer questions. All answers are positive integers. Only submitted answers will be considered for grading.
**No calculators.**

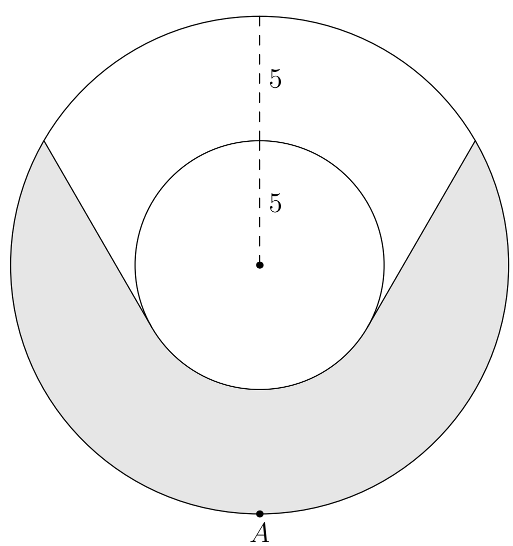
1. A Yule log is shaped like a right cylinder with height 10 and diameter 5. Freya cuts it parallel to its bases into 9 right cylindrical slices. After Freya cut it, the combined surface area of the slices of the Yule log increased by $a\pi$. Compute $a$.

2. Let $O$ be a circle with diameter $AB = 2$. Circles $O_1$ and $O_2$ have centers on $\overline{AB}$ such that $O$ is tangent to $O_1$ at $A$ and to $O_2$ at $B$, and $O_1$ and $O_2$ are externally tangent to each other. The minimum possible value of the sum of the areas of $O_1$ and $O_2$ can be written in the form $\frac{m\pi}{n}$, where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.

3. Right triangular prism $ABCDEF$ with triangular faces $\triangle ABC$ and $\triangle DEF$ and edges $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ has $\angle ABC = 90°$ and $\angle EAB = \angle CAB = 60°$. Given that $AE = 2$, the volume of $ABCDEF$ can be written in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.



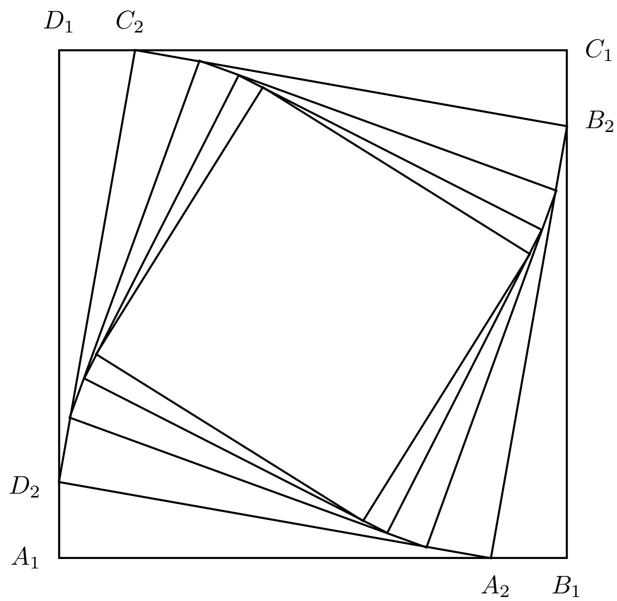

4. Alice is standing on the circumference of a large circular room of radius 10. There is a circular pillar in the center of the room of radius 5 that blocks Alice's view. The total area in the room Alice can see can be expressed in the form $\frac{m\pi}{n} + p\sqrt{q}$, where $m$ and $n$ are relatively prime positive integers and $p$ and $q$ are integers such that $q$ is square-free. Compute $m + n + p + q$. (Note that the pillar is not included in the total area of the room.)



5. Let $A_1 = (0,0), B_1 = (1,0), C_1 = (1,1), D_1 = (0,1)$. For all $i > 1$, we recursively define

$$\begin{aligned} A_i &= \frac{1}{2020}(A_{i-1} + 2019B_{i-1}) \\ B_i &= \frac{1}{2020}(B_{i-1} + 2019C_{i-1}) \\ C_i &= \frac{1}{2020}(C_{i-1} + 2019D_{i-1}) \\ D_i &= \frac{1}{2020}(D_{i-1} + 2019A_{i-1}), \end{aligned}$$

where all operations are done coordinate-wise.




If $[A_iB_iC_iD_i]$ denotes the area of $A_iB_iC_iD_i$, there are positive integers $a, b$, and $c$ such that

$$\sum_{i=1}^{\infty}[A_iB_iC_iD_i] = \frac{a^2b}{c},$$

where $b$ is square-free and $c$ is as small as possible. Compute the value of $a + b + c$.

6. A tetrahedron has four congruent faces, each of which is a triangle with side lengths 6, 5, and 5. If the volume of the tetrahedron is $V$, compute $V^2$.

7. Circle $\Gamma$ has radius 10, center $O$, and diameter $\overline{AB}$. Point $C$ lies on $\Gamma$ such that $AC = 12$. Let $P$ be the circumcenter of $\triangle AOC$. Line $\overleftrightarrow{AP}$ intersects $\Gamma$ at $Q$, where $Q$ is different from $A$. Then the value of $\frac{AP}{AQ}$ can be expressed in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.

8. Let triangle $\triangle ABC$ have $AB = 17$, $BC = 14$, $CA = 12$. Let $M_A, M_B, M_C$ be midpoints of $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ respectively. Let the angle bisectors of $A$, $B$, and $C$ intersect $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ at $P$, $Q$, and $R$, respectively. Reflect $M_A$ about $\overline{AP}$, $M_B$ about $\overline{BQ}$, and $M_C$ about $\overline{CR}$ to obtain $M'_A, M'_B, M'_C$, respectively. The lines $\overleftrightarrow{AM'_A}$, $\overleftrightarrow{BM'_B}$, and $\overleftrightarrow{CM'_C}$ will then intersect $\overline{BC}, \overline{AC}$, and

$\overline{AB}$ at $D$, $E$, and $F$, respectively. Given that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ concur at a point $K$ inside the triangle, in simplest form, the ratio $[KAB] : [KBC] : [KCA]$ can be written in the form $p : q : r$, where $p, q$ and $r$ are relatively prime positive integers and $[XYZ]$ denotes the area of $\triangle XYZ$. Compute $p + q + r$.

9. The *Fibonacci numbers* $F_n$ are defined as $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n > 2$. Let $A$ be the minimum area of a (possibly degenerate) convex polygon with 2020 sides, whose side lengths are the first 2020 Fibonacci numbers $F_1, F_2, \ldots, F_{2020}$ (in any order). A *degenerate convex polygon* is a polygon where all angles are $\leq 180°$. If $A$ can be expressed in the form $\frac{\sqrt{(F_a - b)^2 - c}}{d}$, where $a, b, c$ and $d$ are positive integers, compute the minimal possible value of $a + b + c + d$.

10. Let $E$ be an ellipse where the length of the major axis is 26, the length of the minor axis is 24, and the foci are at points $R$ and $S$. Let $A$ and $B$ be points on the ellipse such that $RASB$ forms a non-degenerate quadrilateral, $\overleftrightarrow{RA}$ and $\overleftrightarrow{SB}$ intersect at $P$ with segment $\overline{PR}$ containing $A$, and $\overleftrightarrow{RB}$ and $\overleftrightarrow{AS}$ intersect at $Q$ with segment $\overline{QR}$ containing $B$. Given that $RA = AS$, $AP = 26$, the perimeter of the non-degenerate quadrilateral $RPSQ$ is $m + \sqrt{n}$, where $m$ and $n$ are integers. Compute $m + n$.