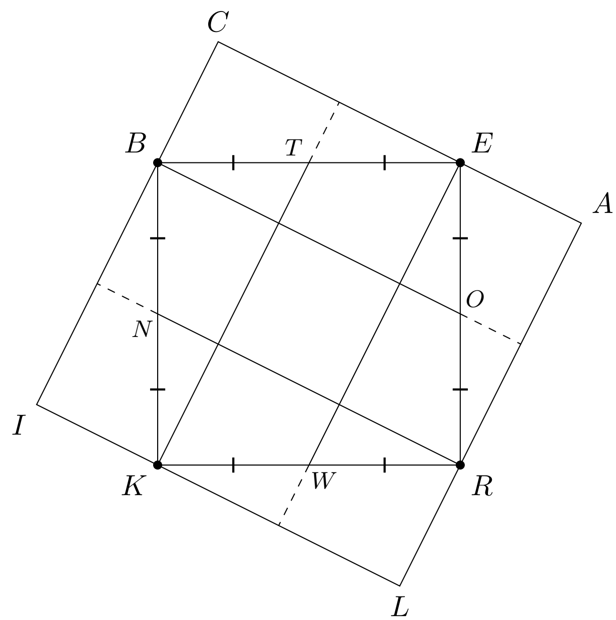
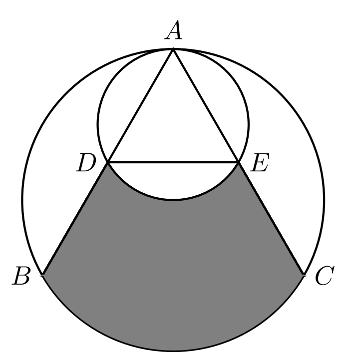
Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. To fold a paper airplane, Austin starts with a square paper $FOLD$ with side length 2. First, he folds corners L and D to the square's center. Then, he folds corner F to corner O. What is the longest distance between two corners of the *resulting* figure?

2. Sohom constructs a square $BERK$ of side length 10. Darlnim adds points T, O, W, and N, which are the midpoints of $\overline{BE}$, $\overline{ER}$, $\overline{RK}$, and $\overline{KB}$, respectively. Lastly, Sylvia constructs square $CALI$ whose edges contain the vertices of $BERK$, such that $\overline{CA}$ is parallel to $\overline{BO}$. Compute the area of $CALI$.



3. Let equilateral triangle $\triangle ABC$ be inscribed in a circle ω_1 with radius 4. Consider another circle ω_2 with radius 2 internally tangent to ω_1 at A. Let ω_2 intersect sides $\overline{AB}$ and $\overline{AC}$ at D and E, respectively, as shown in the diagram. Compute the area of the shaded region.



4. On regular hexagon $GOBEAR$ with side length 2, bears are initially placed at G, B, A, forming an equilateral triangle. At time $t = 0$, all of them move clockwise along the sides of the hexagon at the same pace, stopping once they have each traveled 1 unit. What is the total area swept out by the triangle formed by the three bears during their journey?

5. Steve has a tricycle which has a front wheel with a radius of 30 cm and back wheels with radii of 10 cm and 9 cm. The axle passing through the centers of the back wheels has a length of 40 cm and is perpendicular to both planes containing the wheels. Since the tricycle is tilted, it goes in a circle as Steve pedals. Steve rides the tricycle until it reaches its original position, so that all of the wheels do not slip or leave the ground. The tires trace out concentric circles on the ground, and the radius of the circle the front wheel traces is the average of the radii of the other two traced circles. Compute the total number of degrees the front wheel rotates. (Express your answer in simplest radical form.)

6. Triangle $\triangle BMT$ has $BM = 4$, $BT = 6$, and $MT = 8$. Point A lies on line $\overleftrightarrow{BM}$ and point Y lies on line $\overleftrightarrow{BT}$ such that $\overline{AY}$ is parallel to $\overline{MT}$ and the center of the circle inscribed in triangle $\triangle BAY$ lies on $\overline{MT}$. Compute AY.

7. In triangle $\triangle ABC$ with orthocenter H, the internal angle bisector of $\angle BAC$ intersects $\overline{BC}$ at Y. Given that $AH = 4$, $AY = 6$, and the distance from Y to $\overline{AC}$ is $\sqrt{15}$, compute BC.

8. Anton is playing a game with shapes. He starts with a circle ω_1 of radius 1, and to get a new circle ω_2, he circumscribes a square about ω_1 and then circumscribes circle ω_2 about that square. To get another new circle ω_3, he circumscribes a regular octagon about circle ω_2 and then circumscribes circle ω_3 about that octagon. He continues like this, circumscribing a 2^n-gon about ω_{n-1} and then circumscribing a new circle ω_n about the 2^n-gon. As n increases, the area of ω_n approaches a constant A. Compute A.

9. Seven spheres are situated in space such that no three centers are collinear, no four centers are coplanar, and every pair of spheres intersect each other at more than one point. For every pair of spheres, the plane on which the intersection of the two spheres lies in is drawn. What is the least possible number of sets of four planes that intersect in at least one point?

10. In triangle $\triangle ABC$, E and F are the feet of the altitudes from B to $\overline{AC}$ and C to $\overline{AB}$, respectively. Line $\overleftrightarrow{BC}$ and the line through A tangent to the circumcircle of ABC intersect at X. Let Y be the intersection of line $\overleftrightarrow{EF}$ and the line through A parallel to $\overline{BC}$. If $XB = 4$, $BC = 8$, and $EF = 4\sqrt{3}$, compute XY.